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Holographic Stress Tensor for Kerr-AdS Black Holes and Local Failure on IR-UV Connection

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Abstract

We show that in general holographic stress tensor may contain a new term of divergence of a spacelike unit normal acceleration. Then, it is shown that in contrast to previous descriptions, a new stress tensor for Kerr-AdS solutions can be a traceless one. Interestingly, this prescription entails a local failure on the IR-UV connection.

A precise form of the AdS/CFT correspondence [1] has been formulated by Gubser, Polyakov, Klebanov [2] and Witten [3]. The basic statement is given by the correspondence between the partition function of a string (or M) theory and the generating functional of correlation functions of a boundary conformal field theory (CFT). In the description, the two are functionals of boundary fields $\phi^{(0)}$ which have dual perspective as boundary values of bulk fields ϕ and sources for the operators of CFT. According to this view, in the limit of large number of D-branes and small string coupling, the effective action of strong coupling large N CFT is given by evaluating the action functional for solutions to classical supergravity equations of motion

$$S_{string}[\phi^{cl}(\phi^{(0)})] = W_{CFT}[\phi^{(0)}]. \tag{1}$$

Concentrating on the stress tensor in CFT and the corresponding bulk gauge field, metric $g_{\mu\nu}$, it may be assumed that all other bulk fields vanish on the boundary. For AdS spaces, due to existence of a second order pole on the asymptotic boundary, the metric $g_{\mu\nu}$ does not induce a unique metric $g^{(0)}$ on the boundary. Instead, the boundary field that satisfies the boundary condition of the bulk metric is a conformal structure $[g^{(0)}]$. However, the conformal invariance is to be broken on the regularization of the bulk supergravity action as arbitrarily picking a particular representative $g^{(0)}$ of the conformal structure $[g^{(0)}]$. Taking this scheme, Henningson and Skenderis [4] have shown that the conformal anomaly in CFT (ultraviolet (UV) effect) arises from infrared (IR) divergences in the bulk theory. This is an explicit example of the IR-UV connection [5], which applies to holographic theories [6][7], and becomes a non-trivial check for the conjectured AdS/CFT correspondence. Generalizations and applications of the investigation has been studied in [8][9][10][11][12].

According to the equation (1), divergences arising from the supergravity action is the usual UV divergences in CFT. Thus, the regularization for the supergravity action can be achieved by introducing local counterterms. Compared to the reference background subtraction method [13][14], the prescription, so call counterterm subtraction method, is a nice way to regularize a gravitational action apparently preserving general covariance. The counterterm subtraction method has been developed in its own interest and applications [15][16][17][18][19].

On the other hand, various black holes have been studied in the context of the

AdS/CFT correspondence, and some interesting observations have been made, e.g., on electrically charged Reissner-Nordström-AdS [20], rotating Kerr-AdS [21][22][23], and Kerr-Newman-AdS black holes [24][25]. In this paper, we are concerned about the IR-UV connection between the Kerr-AdS black holes and boundary CFT's living on rotating Einstein universes. This subject has been served by a manuscript [26] in which the correspondence was probed by calculating the Casimir energies and/or conformal anomalies from bulk theory using the counterterm subtraction method.

An interesting observation in [26] has been made for the five-dimensional Kerr-AdS and the dual $\mathcal{N} = 4$ super Yang-Mills (SYM) theory on a rotating Einstein universe in four dimensions. In usual, when a conformal anomaly is present, the classical bulk action contains a logarithmic divergence, which cannot be apparently canceled by a counterterm. For the five-dimensional Kerr-AdS, the stress tensor was not traceless, but logarithmic divergence did not appear in the on-shell action. Nevertheless, the trace of stress tensor precisely matched to that of the dual SYM. The authors argued that not only the integrated conformal anomaly vanishes, but also the anomaly is proportional to $\Box R$, where R is the boundary scalar curvature. Therefore, supplementing ordinary counterterm action with an additional counterterm, then one obtains a new traceless stress tensor. They also proposed that the different choices for counterterms corresponds to the choice of different schemes for regularization in ordinary field theories in four dimensions that is due to the freedom of taking an undetermined coefficient of $\Box R$ in the anomaly.

However, this prescription for the paradox seems unreasonable. First of all, in the one loop effective action of the $\mathcal{N} = 4$ SYM on four-dimensional rotating Einstein universe, the $\Box R$ term in present has to be distinguished from the $\Box R$ term with an undetermined coefficient, e.g., depending on the choices of minimally coupled and conformally coupled scalars [27]. The former is just the usual logarithmic UV divergence in four dimensions $R^{ab}R_{ab} - R^2/3$. The proportionality, $\Box R \propto R^{ab}R_{ab} - R^2/3$, is a special property of the geometry of the four-dimensional rotating Einstein universe[1]. Therefore, it appears that there is not a precise relationship between

[1]Our argument has been given under consideration of the weak coupling calculation. This seems still available in the strong coupling, because the free energy density at weak coupling is only different from that for the strong coupling (and a high temperature limit) up to a constant factor for a leading

addition of new counterterms and the choice of the undetermined coefficient of $\Box R$ in the field theory. Secondly, it has to be noted that the addition of new counterterms means that there may be 'pulsative counterterms' which could be turned on and off depending on given boundary geometries and/or topologies. However, considering the procedure of the derivation of counterterm action in [18], it must be available for all kinds of asymptotic AdS spaces with boundaries of arbitrary geometries and topologies as solutions to the Einstein's equations (containing only the gravitational field without other bulk fields). Thus, it seems hard to put the pulsative counterterms into the counterterm action with an consistent description. One has to do it just by hand.

In this paper, we revisit this paradox. Our starting point is to elaborate on the on-shell action in the context of the ADM formulation. Taking into account this description, we show that in general the stress tensor may contain a new term of divergence of a spacelike unit normal acceleration and be a traceless one. Then we shall discuss that this prescription interestingly entails a local failure on the IR-UV connection; One loop effective action of the $\mathcal{N} = 4$ SYM on four-dimensional rotating Einstein universe is UV finite, and correspondingly the effective action evaluated from bulk action is IR finite. However, the modified stress tensor derived from bulk theory is traceless, while the SYM has non-vanishing trace of stress tensor.

$(d+1)$-dimensional gravitational action with cosmological constant $\Lambda = -d(d-1)/(2\ell^2)$ is given by

$$S = \frac{1}{16\pi G}\int_X d^{d+1}x\sqrt{-g}\left(\hat{R} + \frac{d(d-1)}{\ell^2}\right) - \frac{1}{8\pi G}\int_{\partial X} d^d x\sqrt{-\gamma}\Theta, \tag{2}$$

where ∂X denotes d-dimensional boundary manifold with metric γ_{ab} and Θ_{ab} is the extrinsic curvature of the boundary defined by $\Theta_{ab} = -\gamma_a^\mu \nabla_\mu n_b$. ∇ denotes the covariant derivative on the $(d+1)$-dimensional manifold X and n^μ is an outward unit normal to the boundary. $\hat{R}$ is the bulk scalar curvature. The surface term in (2), so called Gibbons-Hawking term, is required for well defined variational principle. In this paper, we consider the bulk metric of the form

$$g_{\mu\nu}dx^\mu dx^\nu = N^2 dr^2 + \gamma_{ab}dx^a dx^b, \tag{3}$$

term [28][15][29][25].

where x^r is the radial coordinate r, and $N^2 = N^2(r, x^a)$, $\gamma = \gamma(r, x^a)$. In this coordinate system, the spacelike unit normal to the boundary is given by $n_\mu = N\delta^r_\mu$.

According to the counterterm subtraction method, we introduce a counterterm action $\tilde{S}$ regularizing the action (2)

$$\begin{aligned}\tilde{S} &= -\frac{1}{8\pi G}\int_{\partial X} d^d x\sqrt{-\gamma}\left\{\frac{d-1}{\ell} + \frac{\ell}{2(d-2)}R\right.\\ &\left.+\frac{\ell^3}{2(d-2)^2(d-4)}\left(R_{ab}R^{ab} - \frac{d}{4(d-1)}R^2\right) + \cdots\right\}. \end{aligned} \tag{4}$$

Then, the regularized action S_p is defined by $S_p \equiv S + \tilde{S}$. The line element of Kerr-AdS solutions ($d \geq 3$) interested in this paper is [21]

$$\begin{aligned} ds^2 &= -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta d\phi}{\zeta}\right)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{(r^2+a^2)}{\zeta}d\phi\right)^2 \\ &+\frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + r^2\cos^2\theta d\Omega^2_{d-3}, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \rho^2 &= r^2 + a^2\cos^2\theta, \quad \Delta_r = (r^2+a^2)(1+r^2/\ell^2) - 2mGr^{4-d}, \\ \zeta &= 1 - a^2/\ell^2, \quad \Delta_\theta = 1 - (a^2/\ell^2)\cos^2\theta, \end{aligned} \tag{6}$$

and m and a denote the black hole mass and angular momentum per unit mass, respectively. This is an AdS version of higher dimensional Kerr black holes [30].

The on-shell regularized action S^{cl}_p of the five-dimensional Kerr-AdS in (5) does not contain a logarithmic divergence [26]. Divergent part of the on-shell action is given by

$$\begin{aligned} S^{cl}_{div} &= -\frac{1}{8\pi G}\int_X d^{d+1}x\sqrt{-g}\frac{d}{\ell^2} - \frac{1}{8\pi G}\int_{\partial X} d^d x\sqrt{-\gamma}\Theta, \qquad (7)\\ &= \int_{\partial X} d^d x\frac{\sqrt{\Omega_{d-3}}}{8\pi G}r^{d-2}\left[\frac{(d-1)}{\ell^2}r^2 + (d-1)\left(1 + \frac{a^2}{\ell^2}\left(1 - \frac{2\cos^2\theta}{d-2}\right)\right)\right. \\ &\left. -\frac{a^4}{r^4}\cos^2\theta\sin^2\theta\Delta_\theta + \cdots + \frac{a^2(-a^2\cos^2\theta)^{(d-6)/2}}{r^{d-4}}\sin^2\theta\Delta_\theta\right]\frac{\sin\theta\cos^{d-3}\theta}{\zeta}. \end{aligned}$$

In (7) and hereafter, we set $m = 0$ in the metric (5). The terms including the mass is in fact finite on the asymptotic region and is irrelevant to aim of this paper. In addition, it is a necessary condition for counterterms that must be given in terms of only intrinsic boundary geometry.

On the other hand, the divergence structure of the on-shell action is tightly constrained by the Gauss-Codazzi equations in the sense that these are covariant expressions given in terms of the intrinsic and extrinsic boundary geometry [18]. Thus, we expect that the ADM formulation gives us a hint for resolving the paradox above mentioned. In fact, using the ADM formulation, the on-shell action can be expressed by only the intrinsic boundary geometry up to redefinition of the radial coordinate [31]. In this sense, we calculate the on-shell action again in the context of the ADM formulation.

The canonical form of the action (2) is

$$S = \frac{1}{16\pi G}\int_X d^{d+1}xN\sqrt{-\gamma}\left(\Theta^2 - \Theta^{ab}\Theta_{ab} + R + \frac{d(d-1)}{\ell^2}\right). \tag{8}$$

On the other hand, the Einstein equations contracted by the bulk metric can be written by

$$\Theta^2 - \Theta^{ab}\Theta_{ab} - R - \frac{d(d-1)}{\ell^2} = 0, \tag{9}$$

and

$$\Theta^{ab}\Theta_{ab} - n^\mu\nabla_\mu\Theta - \nabla_\mu b^\nu - \frac{d}{\ell^2} = 0, \tag{10}$$

where $b^\mu \equiv n^\nu\nabla_\nu n^\mu$ is the acceleration of the unit normal n^μ. The first equation (9) is the normal-normal component of the Gauss-Codazzi equations [32] and the second (10) can be identified with the tangential-tangential one (requiring the equation (9)). Substituting (9) into (8), the on-shell action is given by

$$S^{cl} = \frac{1}{8\pi G}\int_X d^{d+1}xN\sqrt{-\gamma}\left(R + \frac{d(d-1)}{\ell^2}\right). \tag{11}$$

It must be noted that since we are concerned about the divergence structure of the on-shell action, the equation (10) is irrelevant in our calculation [18]. However, the term of divergence of the acceleration in (10) is to play an important role in our prescription.

Now, we find that divergent part of the on-shell action (11) for d-dimensional Kerr-AdS solutions ($d = 4, 6, \cdots$) contains a logarithmic term

$$\begin{aligned} S^{cl}_{div} &= \int_{\partial X} d^dx\frac{\sqrt{\Omega_{d-3}}}{8\pi G}r^{d-2}\left[\frac{(d-1)}{\ell^2}r^2 + (d-1)\left(1 + \frac{a^2}{\ell^2}\left(1 - \frac{2\cos^2\theta}{d-2}\right)\right)\right. \\ &\quad + \frac{a^2}{r^2}\left(d - 3 + \frac{2((d-3)\sin^2\theta - \cos^2\theta)}{d-4} - \frac{2a^2\cos^2\theta((d-2)\sin^2\theta - \cos^2\theta)}{\ell^2(d-4)}\right) \end{aligned}$$

$$
-\frac{a^4}{r^4}\left(\frac{2\cos^2\theta((d-3)\sin^2\theta-\cos^2\theta)}{d-6}-\frac{2a^2\cos^4\theta((d-2)\sin^2\theta-\cos^2\theta)}{\ell^2(d-6)}\right)
$$
$$
+\cdots+2(-\cos\theta)^{(d-4)}\ln r\left(\frac{a}{r}\right)^{(d-4)}\Big((d-3)\sin^2\theta-\cos^2\theta
$$
$$
-\frac{a^2\cos^2\theta}{\ell^2}((d-2)\sin^2\theta-\cos^2\theta)\Big)\Bigg]\frac{\sin\theta\cos^{d-3}\theta}{\zeta}. \qquad (12)
$$

The logarithmic divergence term in (12) apparently leads a conformal anomaly

$$
\begin{aligned}
\mathcal{A} \;=\; & \frac{-a^2(-a^2\cos^2\theta)^{(d-4)/2}}{8\pi G}\times \\
& \left(\frac{(d-3)\sin^2\theta-\cos^2\theta-a^2\cos^2\theta((d-2)\sin^2\theta-\cos^2\theta)/\ell^2}{\rho r^{d-3}\Delta_r^{(m=0)}}\right), \qquad (13)
\end{aligned}
$$

where we used a cutoff r^2/ℓ^2 (c.f. [17]). As expected, for the case of $d=4$ the conformal anomaly in the leading contribution, $\mathcal{A}_{d=4}$, is equal to that evaluated in [26]

$$
\mathcal{A}_{d=4}=-\frac{a^2\ell}{8\pi G r^4}\left(\frac{a^2\cos^2\theta}{\ell^2}(3\cos^2\theta-2)-\cos 2\theta\right). \qquad (14)
$$

Finally, we are in position of describing the discrepancy of the on-shell actions in (7) and (12). Following the above observation, especially deriving the conformal anomaly (14), the discrepancy should be closely related to the paradox given in [26] why the stress tensor is not traceless, while the on-shell action (7) does not contain a logarithmic divergence.

The origin of the discrepancy is found in the canonical form of the action (8). In fact, it contained two total divergent terms, one canceled the Gibbons-Hawking term, and the other was discarded by a simple algebraic relation given by

$$
\begin{aligned}
S_{bt} &= \frac{1}{8\pi G}\int_X d^{d+1}x\sqrt{-g}\nabla_\mu(n^\nu\nabla_\nu n^\mu) \\
&= \frac{1}{8\pi G}\int_{\partial X} d^dx\sqrt{-\gamma}n_\mu(n^\nu\nabla_\nu n^\mu)=0. \qquad (15)
\end{aligned}
$$

However, it is easily shown that the above calculation is not correct. In the coordinate system of (3), the divergence of acceleration $\nabla_\mu b^\mu$ cannot be a surface term of the ‘timelike’ boundary, because it is not indeed given by a total derivative term of the radial coordinate

$$
\sqrt{-g}\nabla_\mu b^\mu=-\partial_a(\sqrt{-\gamma}\gamma^{ab}\partial_b N)=-\sqrt{-\gamma}D^aD_aN, \qquad (16)
$$

where D_a is the covariant derivative defined on the timelike boundary. Thus, we have to keep this term in calculation of the on-shell action (11)

$$S^{cl} = \frac{1}{8\pi G}\int_X d^{d+1}xN\sqrt{-\gamma}\left(R + \frac{d(d-1)}{\ell^2} + \nabla_\mu b^\mu\right). \tag{17}$$

The on-shell action (17) for Kerr-AdS solutions does not contain the logarithmic divergence term and recover that of (7). Therefore, the stress tensor may be modified by the term divergence of the unit normal acceleration and be a traceless one.

Definition of the stress tensor is

$$T^{ab} \equiv \frac{2}{\sqrt{-\gamma}}\frac{\delta S^{cl}}{\delta\gamma_{ab}} = \frac{1}{8\pi G}\left(\Theta^{ab} - \gamma^{ab}\Theta\right). \tag{18}$$

Actually, in the second equality, it was assumed that $\gamma^{\mu\nu}\delta n_\mu = \delta\gamma^{\mu\nu}n_\mu = 0$. This means that the boundary is fixed under the variations so that the variations of the normal dual-vector on the boundary are proportional to the normal dual-vector. (For an example, see [33].) However, this assumption is no longer proper for the type of metric (3) in which the radial lapse N is a function of a boundary coordinate as well as the radial one, e.g., Kerr-AdS solutions, and this restriction has to be relaxed. Unfortunately, taking the relaxation of the assumption, we cannot calculate the new stress tensor directly, because of a difficulty related to the special algebraic form of the divergence of the acceleration. In this paper, according to the above observations and some physical intuition, we predict a plausible new stress tensor as

$$\begin{aligned} T^{ab}_{new} &= \frac{1}{8\pi G}\left(\Theta^{ab} - \gamma^{ab}\Theta + \frac{1}{2}\gamma^{ab}\int drN\nabla_\mu b^\mu\right) \\ &= \frac{1}{8\pi G}\left(\Theta^{ab} - \gamma^{ab}\Theta - \gamma^{ab}K\right), \end{aligned} \tag{19}$$

where $K \equiv \frac{1}{2}\int drD^cD_cN$. (In the following, we are calling the terms that are just intuitively related to the K term as 'K term'.) We put the K term in (19) such that the new 'physical' stress tensor $T^{ab}_p \equiv T^{ab}_{new} + \tilde{T}^{ab}$ is traceless.

Of cause, the above description in action level just let us know that there must be a contribution of the divergence of acceleration in the stress tensor, and our prediction of the new stress tensor in (19) is due to a usual scheme that the on-shell action does not contain a logarithmic divergence, then the trace of the stress tensor vanishes. However, there is an important reason for the prediction of (19). Kerr black hole solutions, which are asymptotically flat spacetime, can be obtained by taking the flat spacetime limit $\ell \to \infty$. In the case, one can see that the 'old' definition of the stress tensor (18) has a non-vanishing trace in leading contribution, which is the same with the anomaly in (13) in the flat spacetime limit [31]. (In the calculation, the counterterm action for $d = 4$ is the form of

$$\tilde{S} = -\frac{1}{8\pi G}\int_{\partial X} d^4x\sqrt{-\gamma}\sqrt{\frac{3}{2}R}, \tag{20}$$

(For the $d = 4$ case, the counterterm action (20) is enough to eliminate the divergence appearing in the classical action[2].) and the counterterm stress tensor $\tilde{T}^{ab}$ is given by

$$\tilde{T}^{ab} = \frac{1}{8\pi G}\left(\Phi(R^{ab} - \gamma^{ab}R) + D^aD^b\Phi - D^cD_c\Phi\gamma^{ab}\right), \tag{21}$$

where $\Phi \equiv \sqrt{3/(2R)}$.) In order to remedy this problem, the new stress tensor must be given by (19).

Even though the new traceless stress tensor (19) is plausible with the fact that the on-shell action does not contain logarithmic divergence, it seems to be problematic on the AdS/CFT correspondence. As mentioned above, the one loop effective action of the $\mathcal{N} = 4$ SYM on four-dimensional rotating Einstein universe is UV finite, so the equation (1) is still satisfied. However, the SYM has non-vanishing conformal anomaly. In this sense, it appears that *the IR-UV connection is locally broken.* In order to discuss this problem, we need to consider some aspects of the K term in canonical point of view.

In some sense, the K term measures how much deviated the boundary geometry is from a round sphere. This reflects that the contribution appears in the tangential-tangential component of the Gauss-Codazzi equations (10). On the other hand, the canonical form of the action (8) including the K term is written in terms of canonical variables

$$S = \int_X d^{d+1}x\left(\pi^{ab}\gamma'_{ab} - N\mathcal{H} - \frac{\sqrt{-\gamma}}{8\pi G}D^aD_aN\right), \tag{22}$$

where π^{ab} is the conjugate momenta defined by $\pi^{ab} \equiv \delta S/\delta\gamma'_{ab}$. The radial Hamiltonian density $\mathcal{H}$ is given by

$$\mathcal{H} = \frac{16\pi G}{\sqrt{-\gamma}}\left(\frac{\pi^2}{d-1} - \pi_{ab}\pi^{ab}\right) - \frac{\sqrt{-\gamma}}{16\pi G}\left(R + \frac{d(d-1)}{\ell^2}\right). \tag{23}$$

[2]For higher dimensional Kerr solutions, see [18][19]

Now, how can we understand the K term in the canonical action (22)? First of all, the canonical action (22) can be rewritten by

$$S = \int dr \left[\int_{\partial X} d^d x \left(\pi^{ab} \gamma'_{ab} - N\mathcal{H} \right) - \frac{1}{8\pi G} \int_{\partial\partial X} d^{d-1}x \sqrt{|h|} u^a D_a N \right], \qquad (24)$$

where h is an induced metric of a $(d-1)$-dimensional boundary $\partial\partial X$ and u^a is a unit normal to the $\partial\partial X$. Thus, the K term becomes a surface term of the Hamiltonian H

$$H = \int_{\partial X} d^d x \mathcal{H} + \frac{1}{8\pi G} \int_{\partial\partial X} d^{d-1}x \sqrt{|h|} u^a D_a N. \qquad (25)$$

In usual, a surface term of a Hamiltonian plays an important role in physics, e.g., as the total energy of a system. In this paper, we cannot find any physical description for this surface term. According to the AdS/CFT correspondence, the Hamiltonian constraint $\mathcal{H} = 0$ (turning on bulk scalar fields) is equivalent to the renormalization group flow (RG-flow) equation of the boundary CFT [34]. However, the surface term should not give any contribution on the CFT deformation, and moreover, we have been concentrated only on the asymptotic boundary in which bulk scalar fields vanish. Thus, even though it will be find that the surface term in (25) give a kind of (local) deformation of boundary dual CFT related to the local failure on the IR-UV connection, it should be different from the CFT deformations recently studied in the holographic RG-flow (For review, see [35] and references therein). We leave the investigation of prescription for a possible description for the surface term in (25) and its relationship with the local failure on the IR-UV connection as a future work.

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